Mathematical Modeling and Analysis

Efficient Linearity and Bound Preserving Remapping for Meshes with Changing Connectivity

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Conservative interpolation (remapping) is one of the essential parts of most Arbitrary Lagrangian-Eulerian (ALE) methods. It recomputes the conservative quantities (such as mass, momenta, or energy) from the Lagrangian computational mesh to the improved one. In this short report we extend the idea of the swept integration introduced in [1] to the meshes with changing connectivity. We focus to the Voronoi meshes in 2D. One numerical example is presented to show, that properties of this algorithm (conservativity, linearity and bound preservation) remains unchanged for meshes with different topology.

The remapping problem is following – we have two different meshes (original and new) and the unknown underlying function. We do not know the function itself, only mean values in the original cells are known. In [2], we extend idea of [1] to a complete three-step algorithm: 1) reconstruction – recovers the unknown function in a piecewise linear form; 2) integration – integrates the reconstructed function in the new cells to get new mean values; and 3) repair – ensures local-bound preservation by local mass redistribution.

In the reconstruction stage, the slopes in each cell are computed using certain method, with or without limiters. The only condition is to preserve a global linear function. In practical tests, it is convenient to use the monotonicity preserving Barth-Jespersen limiter [3] radically reducing the number of possible local-bound violations.

During the integration stage new mean values are computed. The most natural approach is the exact integration over the overlapping areas of the original and the new meshes. Unfortunately, this process is quite inefficient and makes the complete algorithm very slow. We use the approximate integration method based on the swept region idea – the mass in the new cell can be computed from the mass of the corresponding original cell just by adding or subtracting masses of all swept regions. By swept regions we mean the areas defined by the smooth movement of all cell edges to their new positions. For this algorithm no intersections are needed, moreover it is edge-based and so much more efficient than the previous method.

Unfortunately, our approximate integration does not guarantee satisfaction of the local-bound preservation condition, so the third stage – repair – is needed. It locates the problematic areas, where the bounds are violated, and corrects the value back to the local extreme. Due to the conservativity demand, it takes the mass needed for repair from (or adds additional mass to) the neighboring cells proportionally to the masses, which can safely be taken (added) from these neighbors without violating their local bounds.

This complete algorithm is efficient, linearity and local-bound preserving, stable, and applicable to general unstructured meshes both in 2D [2] and 3D [4] with the same topology. There also is a question, what to do, if the mesh topology changes during the rezoning process. Typical example of reconnection is displayed in the first Figure (a). Four cells of the original mesh changed their topology, one of original edges was removed and a new one added, making neighbors two new cells which have not been neighbors before.

Let us note, that such topology change does not affect the reconstruction and repair process, only the integration stage has to be modified. Our approach is to split the swept integration stage into two steps. We compute the "center of reconnection" – either intersection of removed and created edges or average of their vertices. Then, we shrink the removed edge to this central point and perform swept integration of all five edges involved (the removed edge and four edges connected to its vertices) – see first Figure (b). In

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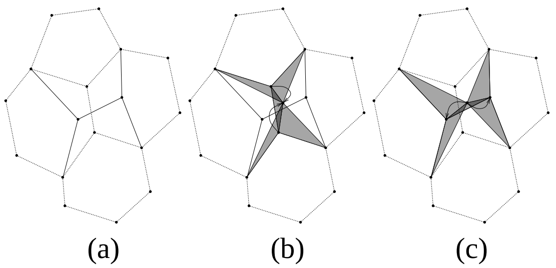
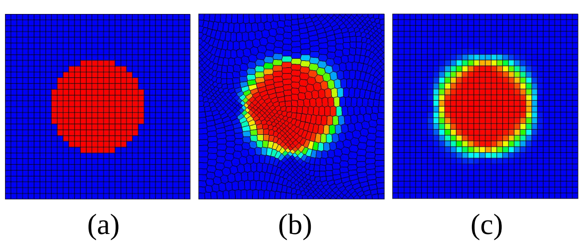


One reconnection in 2D mesh (a). Solving in two steps: shrinking old edges to one point (b) and expanding it to the new edges (c).

the second step, we expand the central point to the created edge and perform similar five swept integrations. The situation at the boundary and the computation of the cell neighborhood for extrema computation must be treated carefully, for the complete algorithm to satisfy all the conditions stated in the previous paragraph.

To demonstrate its properties, we present here the cyclic remapping of the "color function" with value 1 inside a circle in the computational domain center, and 0 outside (second Figure). The



Discontinuous "color function" remapped over a series of sine-moving Voronoi grids. Initial values (a), values on the middle mesh (b), and values on the final mesh (c).

initial Voronoi mesh (a) has regularly placed generators, and the n-th mesh (in time $t^n = n/n_{\max}$) is obtained from generators placed by formulas

$$x_k^n = x_k^0 + \alpha(t^n)\,\sin(2\pi x_k^0)\,\sin(2\pi y_k^0)\,,$$
$$y_k^n = y_k^0 + \alpha(t^n)\,\sin(2\pi x_k^0)\,\sin(2\pi y_k^0)\,,$$
$$\alpha(t) = \begin{cases} t/5 & \text{for} \quad t \le 1/2 \\ (1-t)/5 & \text{for} \quad t > 1/2\,. \end{cases}$$

During the generator's movement, the grid moves and changes it's connectivity up to the middle of the simulation (b), then it returns back to the original one (c).

The performed simulations and their numerical errors prove the algorithm's convergence and satisfaction of all required properties for the quality remapping algorithm.

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